Chapter 23

RISK MANAGEMENT*

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Abstract

Even though risk management is the quality control of finance to ensure the smooth functioning of the business model and the corporate model, this chapter takes a more focused approach to risk management. We begin by describing the methods to calculate risk measures. We then describe how these risk measures may be reported. Reporting provides feedback to the identification and measurements of risks. Reporting enables the risk management to monitor the enterprise risk exposures so that the firm has a built-in, self-correcting procedure that enables the enterprise to improve and adapt to changes. In other words, risk management is concerned with four different phases, which are risk measurement, risk reporting, risk monitoring, and risk management in a narrow sense. We focus on risk measurement by taking a numerical example. We explain three different methodologies for that purpose, and examine whether the measured risk is appropriate based on observed market data.

Keywords: value at risk; market risk; delta-normal methodology; delta-gamma methodology; volatility; component VaR; historical simulation; Monte Carlo simulation; back testing; risk reporting

In recent years, a subject called risk management quickly established an indispensable position in finance, which would not surprise us, because finance has studied how to deal with risk and we have experienced many catastrophic financial accidents resulting in much loss such as Orange County and Long Term Capital Management.

Risk management as a broad concept consists of four phases: risk measurement, risk reporting, risk monitoring, and risk management in a narrow sense. We will discuss the four phases one by one mainly focusing on risk measurement.

23.1. Risk Measurement

Risk measurement begins with identifying all the sources of risks, and how they behave in terms of the probability distribution, and how they are manifested. Often, these sources of risk are classified as market risk, credit risk, liquidity risk, and legal risk. More recently, there are operational risks and business risks.

23.1.1. Market Risk

Market risk is often defined as the losses that arise from the mark to market of the trading securities. These trading securities may be derivatives such as swaps, swaptions, caps, and floors. They can be securities such as stocks and bonds. Market risk is referred to as the potential loss of the portfolio due to market movements.

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While this is the basic idea of the market risk, the measure of the "value" is a subject of concern. Market risk is concerned with the fall in the mark to market value. For an actively trading portfolio that is managed at a trading desk, the value is defined as the sell price of the portfolio at normal market conditions. For this reason, traders need to mark their portfolio at their bid price at the end of the trading day, the mark to market value. Traders often estimate these prices based on their discussions with counter-parties, or they can get the prices from market trading systems.

We need to extend the mark to market concept to determine the risk measure, which is the potential loss as measured by the "mark to market" approach.

23.1.2. Value at Risk (VaR)

To measure the risks, one widely used measure is the Value at Risk (VaR). So far, risk in finance has been measured depending on which securities we are concerned with. For example, beta and duration have been the risk measures for stocks and bonds, respectively. The problem with this approach is that we cannot compare the stock's risk with the bond's risk. To remedy this drawback, we need a unified measure for comparison purposes, which has prompted the birth of VaR risk measure. Value at Risk is a measure of potential loss at a level (99 percent or 95 percent confidence level) over a time horizon, say, 7 days. Specifically 95 percent-1-day-VaR is the dollar value such that the probability of a loss for 1 day exceeding this amount is equal to 5 percent. For example, consider a portfolio of $100 million equity. The annualized volatility of the returns is 20 percent. The VaR of the portfolio over 1 year is $46.527 million (i.e.100–53.473) and $32.8971 million (i.e.100–67.1029), for 99 percent or 95 percent confidence levels, respectively. If we imagine a normal distribution which has a mean of $100 million and a 20 percent standard deviation, the probabilities that the normally distributed variable has less than $53.473 million and $67.1029 million are 1 percent and 5 percent, respectively. In other words, the probability of exceeding the loss of $46.527 million over a 1-year period is 1 percent when the current portfolio value is $100 million, and the annualized volatility of the returns is 20 percent. Therefore, we have a loss exceeding $46.527 million only once out of 100 trials. A critical assumption to calculate VaR here is that the portfolio value follows a normal distribution, which is sometimes hard to accept.

The risk management of financial institutions measures this downside risk to detect potential loss in their portfolio. The measure of risk is often measured by the standard deviation or the volatility. A measure of variation is not sufficient because many securities exhibit a bias toward the upside (profit), as in an option, or the downside (loss), as in a high-yield bond, which is referred to as a skewed distribution, as compared to a symmetric distribution such as a normal distribution. These securities do not have their profits and losses evenly distributed around their mean. Therefore the variation as a statistic would not be able to capture the risk of a position. Volatility is a measure of variability, and may not correctly measure the potential significant losses of a risky position.

VaR has gained broad acceptance by regulators, investors, and management of firms in recent years because it is expressed in dollars, and consistently calculates the risk arising from the short or long positions and different securities. An advantage of expressing VaR in dollars is that we can compare or combine risk across different securities. For example, we have traditionally denoted risk of a stock by beta and risk of a bond by duration. However, if they have different units in measuring the stock and the bond, it is hard to compare the risk of the stock with that of the bond, which is not the case in VaR.

There are three main methodologies to calculate the VaR values: Delta-normal methods, Historical simulation, and the Monte Carlo simulation.

23.1.2.1. Delta-Normal Methodology

The delta-normal methodology assumes that all the risk sources follow normal distributions and the VaR is determined assuming that the small change of the risk source would lead to a directly proportional small change of the security's price over a certain time horizon.

VaR for single securities: Consider a stock. The delta-normal approach assumes that the stock price itself is the risk source and it follows a normal distribution. Therefore, the uncertainty of the stock value over a time horizon is simply the annual standard deviation of the stock volatility adjusted by a time factor. A critical value is used to specify the confidence level required by the VaR measure. Specifically, the VaR is given by:

$$\text{VaR} = \alpha \times \text{time factor} \times \text{volatility} \tag{23.1}$$

α is called the critical value, which determines the one-tail confidence level of standard normal distribution. Formally, α is the value such that the confidence level is equal to the probability that X is greater than α, where X is a random variable of a standard normal distribution.

Time factor is defined as $\sqrt{t}$, where t is the time horizon in measuring the VaR. The time-measurement unit of the time factor should be consistent with that of the volatility. For example, if the volatility is measured in years, t is also measured in years.

Volatility is the standard deviation of the stock measured in dollars over 1 year.

The problem for a portfolio of stocks is somewhat more complicated. In principle, we can use a large matrix of correlation of all the stock returns, and calculate the value. In practice, often this is too cumbersome. The reason for this is that, since we treat each stock as a different risk source, we have the same number of risk sources as that of the stocks constituting the portfolio. For example, if we have a portfolio consisting of 10 stocks, we have to estimate 10 variances and 45 co-variances. One way to circumvent it is to use the Capital Asset Pricing Model. Then the portfolio return distribution is given by:

$$E[R_P] = r_f + \beta_P(E[R_M] - r_f). \tag{23.2}$$

The distribution of the portfolio is therefore proportional to the market index by a beta. By using the CAPM, we have only one risk source regardless of the size of a portfolio, which makes it much simpler to calculate portfolio VaR.

The VaR calculation for bonds requires an extra step in the calculation. The risk sources for default-free bonds are interest rate risks. These risks, per se, do not directly measure the loss. In the case of stocks, the fall in stock price is the loss. But for bonds, we need to link the rise in interest rates to the loss in dollar terms.

By the definition of duration, we have the following equation

$$\Delta P = -\$\text{Duration} \times \Delta r, \tag{23.3}$$

where \$Duration is the dollar duration defined as the product of the price and duration.

$$\$\text{Duration} = P \times \text{Duration}. \tag{23.4}$$

Δr is the uncertain change in interest rates over the time horizon for the VaR measure. We assume that this uncertain movement has a normal distribution with zero mean and standard deviation σ. The interest rate risk is described by a normal distribution. For the time being, we assume that the interest rate risk is modeled by the uncertain parallel movements of the spot-yield curve and the yield curve is flat at r.

Given these assumptions, it follows from Equation (23.3) that the price of the bond, or a bond position, has a normal distribution given by:

$$\widetilde{\Delta} P = -\$\text{Duration} \times \widetilde{\Delta} r$$

The means of calculating the critical value for a particular interval of a normal distribution is therefore given by:

$$\text{VaR(bond)} = \alpha \times \text{time factor} \times \$\text{Duration} \times \sigma \times r$$

$$\sigma = \text{SD}\left(\frac{\Delta r}{r}\right) \tag{23.5}$$

Since the standard deviation in Equation (23.5) is based on a proportional change of interest rates, we should multiply by r to get the standard deviation of a change of interest rates.

The above formula assumes that the spot-yield curve makes a parallel shift movement and is flat, because \$duration is derived based on the same assumptions. Further, the above formula assumes that the uncertain changes in interest rates follow a normal distribution, because we use the standard deviation to measure risk. More generally, we can assume that the yield curve movements are determined by n key rates $r(1), r(2), \ldots, r(n)$. These key rate uncertain movements are assumed to have a multivariate normal distribution over the time horizon t of the VaR measure with the variance–covariance Ω. Given this multiple risk factor model, the bond price uncertain value is a multivariate normal distribution given by:

$$\widetilde{\Delta}P = -\sum_{i=1}^{n} \$\mathrm{KRD}(i)\,\widetilde{\Delta}r(i),$$

where \$KRD(i) is the dollar key rate duration given by the $P \times \mathrm{KRD}(i)$. KRD(i) is the key rate duration. It is the bond price sensitivity to the ith key rate movement. Then it follows that the VaR of the bond is given by:

$$\mathrm{VaR(bond)} = \alpha \times \text{time factor} \times \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \$\mathrm{KRD}(i)\,\$\mathrm{KRD}(j)\,\Omega_{ij} \right)^{0.5}, \qquad (23.6)$$

where the dollar key rate durations of the bond are denoted by \$KRD. P is the bond price, or the value of the bond position. Ω_{ij} is the ith and jth entry of the variance–covariance matrix Ω, i.e. it is the covariance of the distribution of the ith and jth key rate movements. Here, we calculate the variance–covariance of key rates. Therefore, we do not have to multiply by r.

VaR for a Portfolio: Now, we are in the position to determine the VaR of a portfolio of these types of assets. Suppose the portfolio has n securities. Let P_i be the price of the ith security, which may be the bond price or a stock price. Let x_i be the number of the securities in the portfolio. Then the portfolio value is given by:

$$P = \sum_{i=1}^{n} x_i \cdot P_i. \qquad (23.7)$$

The risk of the portfolio may be measured by the VaR of the portfolio value as defined by Equation (23.7). Let $\Delta\theta_i$ for $i = 1 \ldots n$ be the risk sources, with Ω the variance–covariance of these risks. Let \$Duration(i) be the dollar duration (or sensitivity) of the portfolio to each risk source $\Delta\theta_i$. The portfolio uncertain value is given by:

$$\widetilde{\Delta}P = -\sum_{i=1}^{n} \$\mathrm{Duration}(i)\,\widetilde{\Delta}\theta_i, \qquad (23.8)$$

where P is the portfolio value. Following the above argument, the VaR of the portfolio is given by:

$$\mathrm{VaR(portfolio)} = \alpha \times \text{time factor} \times \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \$\mathrm{Duration}(i)\,\$\mathrm{Duration}(j)\,\Omega_{ij} \right)^{0.5} \qquad (23.9)$$

We can now calculate the contribution of risk for each risk source to the portfolio VaR. Let us define VaR β_i (also called the component VaR) to the ith risk source θ_i to be:

$$\mathrm{VaR}\ \beta_i(\mathrm{portfolio}) = \alpha \times \text{time factor} \times \sum_{j=1}^{n} \$\mathrm{Duration}(i)\,\$\mathrm{Duration}(j)\,\Omega_{ij} \times \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \$\mathrm{Duration}(i)\,\$\mathrm{Duration}(j)\,\Omega_{ij} \right)^{-0.5}$$

VaR β_i is the contribution of risk by ith risk source to the VaR measure. It is clear from the definition that

$$\sum_{i=1}^{n} \mathrm{VaR}\ \beta_i = \mathrm{VaR} \qquad (23.10)$$

This means the sum of the component VaR (VaR β_i) is equal to the VaR of the portfolio.

Since the risk sources are correlated with each other, we have to appropriately identify the effect of correlations and diversifications on the risks to measure the risk contribution of each risk source to the VaR of the portfolio. VaR β_i is a way to isolate all these effects.

A Numerical Example: To calculate the VaR of a portfolio of three different stocks (GE, CITI, and HP), we calculate the daily rate of returns for each stock and estimate the variance–covariance matrix of the stocks' returns. The sample period is from January 3, 2001 to May 2, 2002. The number of total observations is 332. For the purpose of calculating VaR, we assume that the expected proportional changes in the stock prices over 1 day are equal to 0. To calculate the daily rates of return and the variance–covariance matrix, we use the following formulas:

$$
\begin{aligned}
r_{i,t} &= \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}}, \quad \forall i = \text{GE, CITI, and HP} \\
\bar{r}_i &= 0 \\
\sigma_i^2 &= \frac{1}{m}\sum_{t=1}^{m}(r_{i,t} - \bar{r}_i)^2 \\
\sigma_{i,j} &= \frac{1}{m}\sum_{t=1}^{m}(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_i),
\end{aligned}
$$

where m is the number of days in the estimation period.

We first calculate the individual stock VaR, and then the stock portfolio VaR to measure the diversification effect. We assume the size of the portfolio position to be $100 and the invested weights to be equal. Further, we assume that the significance level is 1 percent and the horizon period is 5 days.

First, we calculate the variance–covariance matrix assuming that the expected means are 0. From the variance–covariance matrix, we can get standard deviations of each individual stock as well as the standard deviation of the portfolio with equal weights. To get the standard deviation of the portfolio, we premultiply and postmultiply the variance–covariance matrix with the weight vector. The variance–covariance matrix $\mathbf{\Omega}$, the correlation matrix $\mathbf{\Sigma}$ of three stocks, and the variance of the portfolio consisting of three stocks are given below.

$$
\mathbf{\Omega} = \begin{pmatrix} 0.00060272 & 0.00038256 & 0.00034470 \\ 0.00038256 & 0.00047637 & 0.00032078 \\ 0.00034470 & 0.00032078 & 0.00126925 \end{pmatrix}
$$

$$
\mathbf{\Sigma} = \begin{pmatrix} 1.00000000 & 0.71396050 & 0.39410390 \\ 0.71396050 & 1.00000000 & 0.41253223 \\ 0.39410390 & 0.41253223 & 1.00000000 \end{pmatrix}
$$

$$
\mathbf{w}^{\mathbf{T}} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)
$$

$$
\begin{aligned}
\sigma^2_{\text{Portfolio}} &= \mathbf{w}^{\mathbf{T}}\mathbf{\Omega}\mathbf{w} = (1/3\ 1/3\ 1/3) \\
&\begin{pmatrix} 0.00060272 & 0.00038256 & 0.00034470 \\ 0.00038256 & 0.00047637 & 0.00032078 \\ 0.00034470 & 0.00032078 & 0.00126925 \end{pmatrix}\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} \\
&= 0.00049382
\end{aligned}
$$

Second, since we have the equal weight portfolio, the amount that has been invested in each individual stock is 33.33 dollars. Furthermore, since the significance level is assumed to be 1 percent, $\alpha = 2.32635$.

The detailed derivation of the individual VaR as well as the portfolio **VaR** is given as follows.

$$
\begin{aligned}
\text{VaR}_i &= \text{total invest} \times w_i \times \sigma_i \times \alpha \times \sqrt{\text{days}} \\
\text{VaR}_P &= \text{total invest} \times \sigma_P \times \alpha \times \sqrt{\text{days}},
\end{aligned}
\qquad (23.11)
$$

where

$$
\begin{aligned}
i &= \{\text{GE, CITI, HP}\} \\
\sigma_P &= \sqrt{\mathbf{w}^{\mathbf{T}}\mathbf{\Omega}\mathbf{w}} \\
&= \sqrt{\sum_i \sum_j \omega_i \omega_j \sigma_{i,j}} \\
&= \sqrt{\sum_i \omega_i^2 \sigma_i^2 + 2\sum_i \sum_{j \neq i} \omega_i \omega_j \sigma_{i,j}}
\end{aligned}
$$

By plugging the appropriate numbers in Equation (23.11), we can get three individual stock VaRs and the portfolio VaR.

$$\text{VaR}_{GE} = \text{total invest} \times w_{GE} \times \sigma_{GE} \times \alpha \times \sqrt{\text{days}} = \frac{100}{3} \times \sqrt{0.00060272} \times 2.32635 \times \sqrt{5} = 4.25693$$

$$\text{VaR}_{CITI} = \text{total invest} \times w_{CITI} \times \sigma_{CITI} \times \alpha \times \sqrt{\text{days}} = \frac{100}{3} \times \sqrt{0.00047637} \times 2.32635 \times \sqrt{5} = 3.78451$$

$$\text{VaR}_{HP} = \text{total invest} \times w_{HP} \times \sigma_{HP} \times \alpha \times \sqrt{\text{days}} = \frac{100}{3} \times \sqrt{0.00126925} \times 2.32635 \times \sqrt{5} = 6.17749$$

$$\text{VaR}_{P} = \text{total invest} \times \sigma_{P} \times \alpha \times \sqrt{\text{days}} = 100 \times \sqrt{0.00049382} \times 2.32635 \times \sqrt{5} = 11.55968$$

Once we have calculated the VaRs, we are concerned with how much each individual stock contributes to the portfolio risk. To this end, we calculate the betas of individual stocks. We define the beta of the stock here taking the portfolio as "market portfolio" of the CAPM. The method of determining the beta (the systematic risk) of a stock within the portfolio is given by the formula below. The numerator is the covariance of each stock with the market portfolio and the denominator is the variance of the market portfolio, which is the variance of the portfolio consisting of GE, CITI and HP.

$$\text{Beta}_{\text{Delta-Normal Method}} = \begin{pmatrix} \beta_{GE} \\ \beta_{CITI} \\ \beta_{HP} \end{pmatrix} = \frac{\mathbf{\Omega w}}{\mathbf{w^T \Omega w}} = \frac{\mathbf{\Omega} \cdot \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}}{(1/3\ 1/3\ 1/3) \cdot \mathbf{\Omega} \cdot \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}} = \begin{pmatrix} 0.89775 \\ 0.79631 \\ 1.30595 \end{pmatrix}.$$

Component VaR is a product of three parts, which are weight ω_i, β_i, and portfolio VaR. The reason to get the β is that β represents the systematic risk or the marginal contribution of each stock's risk to the portfolio risk.

$$\text{Component VaR}_{i} = \omega_i \times \beta_i \times \text{VaR}_{\text{Portfolio}} \ \forall i = \text{GE, CITI, and HP}$$

For example, the GE component VaR is that

$$\text{Component VaR}_{GE} = \omega_{GE} \times \beta_{GE} \times \text{VaR}_{\text{Portfolio}} = \frac{1}{3} \times 0.89775 \times 11.55968 = 3.45922$$

Since the component VaR is the individual stock's contribution to the portfolio risk, the sum of three component VaRs should be the portfolio VaR. Mathematically, since the sum of each beta multiplied by its corresponding weight is equal to 1, the sum of three component VaRs should be the portfolio VaR.

The final results have been summarized in Table 23.1.

Portfolio effect is defined as the individual stock VaR net of the component VaR, measuring the effect of diversification on the risk of the individual asset risk. When there are many uncorrelated assets in the portfolio, then portfolio effect can be significant. The portfolio effect can also measure the hedging effect within the portfolio if one asset has a negative correlation to another asset.

The advantage of the methodology above is its simplicity; it exploits the properties of a normal

Table 23.1. VaR calculation output by delta-normal method

5-day VaR	GE	CITI	HP	Total
Weight	1/3	1/3	1/3	1
Individual stock VaR	4.25693	3.78451	6.17749	14.21893
Portfolio VaR	–	–	–	11.55968
Beta	0.89775	0.79631	1.30595	–
Beta×Weight	0.29925	0.26544	0.43532	1
Component VaR	3.45922	3.06835	5.03212	11.55968
Portfolio Effects	0.79771	0.71616	1.14537	2.65924

distribution. Specifically, we can use the additive property of the distribution. In doing so, we can build up the VaR of a portfolio from each single security and we can aggregate the information. Finally, we can calculate the contribution of the risk of each security to the portfolio risks. However, the simplicity comes with a cost.

The main drawback is that the normality assumption precludes other distributions that have skewed distribution as the main source of risks. For example, a short position of a call or put option would be misleading with the use of the delta-normal methodology, because the distribution is not normal and the potential losses are much higher than assuming the normal distribution when the time horizon is not sufficiently short. One way to ameliorate the problem is to extend the methodology to incorporate skewness in the measurement. It is important to point out that if security returns are highly skewed (e.g. out of the money options), there will be significant model risks in valuing the securities. In those situations, the error from a delta-normal methodology is only part of the error in the estimation. For this reason, in practice, those securities usually have to be analyzed separately in more detail and they require specific methodologies in managing their risks. Another problem of the normality assumption is the fat-tail effect of stocks, where there is a significant probability for the stock to realize high or low returns. Kurtosis of the stock returns, a measure of the fatness of the tails, is empirically significant.

Another drawback of the delta-normal method comes from the assumption that the risk is measured by the first derivative called delta. When we cannot adequately measure the risk by the first derivative, we should extend to the second derivative called gamma to measure the risk. This method is called the delta-gamma methodology.

However, for the most part, delta-normal does provide a measure of risks enabling risk managers to evaluate the risks of a portfolio.

23.1.4. Historical Simulation Methodology

Historical simulation is another VaR measuring methodology. The method uses a historical period of observed movement of the risk sources: stock returns, interest rate shifts, and foreign exchange rate changes. It simulates the portfolio returns over that period, as if the portfolio were held unchanged over that period of time. The VaR of the portfolio returns is then computed.

This is a simple methodology, particularly for trading desks. The reason is that for most trading desks; the trading books have to be marked to market daily. The modeling technologies are in place to value the securities and aggregate the reports. Simulating the historical scenarios is a fairly straightforward procedure. As in Figure 23.1, we sort the historical return data in an increasing order and locate $x$percent percentile to calculate VaR.

Using the historical return data set of each of the stocks, in Table 23.2, we can find α percent percentile value of their daily returns to calculate the VaR of each stock and portfolio. We also use their historical returns to determine their variance and covariance matrix. With the estimation of this variance and covariance matrix, we can then determine the securities's beta and the component VaR [1]. The results are summarized in Table 23.3.

In comparing Tables 23.1 and 23.3, the results suggest that the two methods do not provide the same VaR numbers, but they are reasonably close

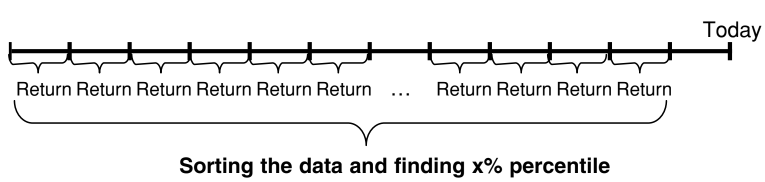

Figure 23.1. The historical simulation methodology

Table 23.2. Historical return data set

Date	(1) GE	(2) CITI	(3) HP	(1)+(2)+(3) Portfolio
2001,01,03	3.0933	2.9307	4.1983	10.2224
2001,01,04	0.1743	0.4550	0.5578	1.1872
2001,01,05	−0.5202	−1.1971	−3.8599	−5.5771
2001,01,08	−1.2330	−0.1925	0.8165	−0.6090
2001,10,29	−1.2431	−1.4958	−0.8403	−3.5793
2001,10,30	−0.9707	−0.6106	−0.8238	−2.4051
2001,10,31	0.0642	−0.0220	−0.2750	−0.2327
2002,04,30	0.7563	0.3265	0.2554	1.3382
2002,05,01	0.1585	0.5081	−0.4678	0.1987
2002,05,02	−0.1052	0.7507	0.4547	1.1003
1% percentile	−4.88495	−4.05485	−6.60260	−12.47086
1% VaR	4.88495	4.05485	6.60260	12.47086 [a]

[a]12.47086 is not equal to the sum of three numbers (4.88495, 4.05485, 6.60260) because of the diversification effect.

Table 23.3. VaR calculation output by historical simulation method

5-day VaR	GE	CITI	HP	Total
Weight	1/3	1/3	1/3	1
Individual stock VaR	4.88495	4.05485	6.60260	15.54241
Portfolio VaR	–	–	–	12.47086
Beta	0.89775	0.79631	1.30595	–
Beta*Weight	0.29925	0.26544	0.43532	1
Component VaR	3.73188	3.31021	5.42877	12.47086
Portfolio Effects	1.15306	0.74465	1.17384	3.07155

within 10 percent error. One source of error can be the normality distribution assumption. To the extent that in the sample period, the stock returns exhibited significant fat-tail behavior, then the discrepancies between the two measures can be significant.

23.1.5. Monte Carlo Simulation Methodology

The Monte Carlo simulation refers to a methodology, where we randomly generate many scenarios and calculate the VaR of the portfolio. The method is similar to the historical simulation method, but the difference is that we now simulate many scenarios using a forward-looking estimate of volatilities and not the historical volatilities over a period of time.

We use a multivariate normal distribution with the given variance–covariance matrix based on the delta-normal method and zero means of the stocks to simulate the stock returns 100,000 times. These returns are then used to calculate the VaR of each stock and the VaR of the portfolio. The variance–covariance matrix of stock returns generated by Monte Carlo simulation is as follows:

$$\boldsymbol{\Omega}_{\text{Monte Carlo}} = \begin{pmatrix} 0.00139246 & 0.00130640 & 0.00156568 \\ 0.00130640 & 0.00124862 & 0.00148949 \\ 0.00156568 & 0.00148949 & 0.00207135 \end{pmatrix}$$

$$\begin{aligned} \text{Monte Carlo VaR}_{\text{GE}} &= 0.01 \text{ Percentile of Scenario}_{\text{GE}} \\ &\quad \times \text{total invest} \times w_{\text{GE}} \times \sqrt{\text{day}} \\ &= 0.08577711 \times \frac{100}{3} \times \sqrt{5} \\ &= 6.39345 \end{aligned}$$

$$\begin{aligned} \text{Monte Carlo VaR}_{\text{CITI}} &= 0.01 \text{ Percentile of Scenario}_{\text{CITI}} \\ &\quad \times \text{total invest} \times w_{\text{CITI}} \times \sqrt{\text{day}} \\ &= 0.08126864 \times \frac{100}{3} \times \sqrt{5} \\ &= 6.05741 \end{aligned}$$

$$\begin{aligned} \text{Monte Carlo VaR}_{\text{HP}} &= 0.01 \text{Percentile of Scenario}_{\text{HP}} \\ &\quad \times \text{total invest} \times w_{\text{HP}} \times \sqrt{\text{day}} \\ &= 0.11359961 \times \frac{100}{3} \times \sqrt{5} \\ &= 8.46722 \end{aligned}$$

$$\begin{aligned} \text{Monte Carlo VaR}_{P} &= 0.01 \text{ Percentile of Scenario}_{P} \\ &\quad \times \text{total invest} \times \sqrt{\text{day}} \\ &= 0.09362381 \times 100 \times \sqrt{5} \\ &= 20.93492. \end{aligned}$$

Using the variance and covariance matrix of the stocks, which we can calculate from the randomly generated returns, we can then determine the component VaR as we have done in the examples above. VaR by the Monte Carlo Simulation Method is given in Table 23.4.

Table 23.4. VaR calculation output by Monte Carlo simulation method

5-day VaR	GE	CITI	HP	Total
Weight	1/3	1/3	1/3	1
Individual stock VaR	6.39345	6.05741	8.46722	20.91807
Portfolio VaR	–	–	–	20.93492
Beta	0.95222	0.90309	1.14469	–
Beta*Weight	0.31741	0.30103	0.38156	1
Component VaR	6.64489	6.30204	7.98799	20.93492
Portfolio Effects	−0.25144	−0.24464	0.47923	−0.01685

The results show that the VaR numbers are similar in all three approaches. This is not too surprising, since the three examples use the same model assumptions: the variance–covariance matrix of the stocks. Their differences result from the use of normality in the delta-normal and the Monte Carlo simulation approaches, whereas the historical simulation is based on the historical behavior of the stocks. Note that while we use the assumption of multivariate normal distributions of the stock in the Monte Carlo example here, in general this assumption is not required, and we can use a multivariate distribution that models the actual stock returns behavior best. Another source of error in this comparison is the model risks. The number of trials in both the historical simulation and the Monte Carlo simulations may not be sufficient for the results to converge to the underlying variances of the stocks.

23.2. Risk Reporting

The sections above describe the measurement of VaR. We can now report the risk exposure and we illustrate it with a bank's balance sheet below[2]. VaR is defined in this report with 99 percent confidence level over a 1-month time horizon.

The report shows the market value (or the fair value) of each item on a bank's balance sheet and the VaR value of each item. VaR/MV is the ratio of VaR to the market value, measuring the risk per dollar, and VaR β_i is the marginal risk of each item to the VaR of the bank (the VaR of the equity).

Table 23.5. VaR table: Aggregation of risks to equity ($ million.)

Items	Market value	VaR	VaR/MV (%)	Component VaR
Prime rate loans	3,286	11.31	0.34	4.5
Base rate loans	2,170	4.92	0.23	−4.3
Variable rate mortgages	625	5.47	0.87	−4.8
Fixed-rate loans	1,231	30.49	2.50	−22.5
Bonds	2,854	33.46	1.17	−28.2
Base-rate time deposits	1,959	5.83	0.30	3.24
Prime-rate time deposits	289	1.56	0.54	0.98
Fixed-rate time deposits	443	11.69	2.64	9.55
Demand deposits	5,250	44.62	0.85	36.89
Long-term market funding	1,146	19.85	1.73	15.16
Equity	1,078	10.59	0.98	10.59

Note that the sum of the VaR values of all the items is not the same as the VaR of the equity. This is because the sum of the VaR values does not take diversification or hedging effects into account. However, the sum of the component VaR is equal to the VaR of the equity, because the component VaR has already reflected the diversification effect or hedging effects. VaR/MV measures the risk of each item per dollar. The results show that the fixed rate loans and the fixed rate time deposits are the most risky with the VaR per dollar being 2.5 percent and 2.64 percent respectively.

The results of the component VaR show that the demand deposit, while not the most risky item on the balance sheet, contributes much of the risk to equity. All the items on the asset side of the balance sheet (except for the prime rate loans) become hedging instruments to the demand deposit position.

One application of this overview of risks at the aggregated and disaggregated level is that we can identify the "natural hedges" in the portfolio. The risk contribution can be negative. This occurs when there is one position of stocks or bonds that is the main risk contributor. Then any security that is negatively correlated with that position would lower the portfolio total risk. The report will show that the risk contribution is negative, and that security is considered to offer a natural hedge to the portfolio. This methodology can extend from a portfolio of securities to a portfolio of business units. These units may be trading desks, a fund of funds, or multiple strategies of a hedge fund.

23.3. Risk Monitoring: *Back testing*[3]

The purpose of the back testing is to see whether the methods to calculate VaR are appropriate in the sense that the actual maximum loss has exceeded the predetermined VaR within an expected margin. The expected margin depends on which significance level we select when we calculate the VaR.

The basic idea behind the back test is to compare the actual days when the actual loss exceeds the VaR with the expected days, based on the significance level. We calculate the expected number of VaR violation days and actual VaR violation days.

23.4. Risk Management

In the previous sections, we have discussed the risk measurement, reporting, and monitoring. Now, we discuss the actions that we can take in managing the risks.

Much of the impetus of risk management started in the aftermath of the series of financial debacles for some funds, banks, and municipalities. In a few years, much progress has been made in research and development. More financial institutions have put in place a risk management team and technologies, including VaR calculations for the trading desks and the firm's balance sheets.

In reviewing the methodologies and technologies developed in these years, one cannot help noticing that most risk management measures and techniques focus on banks and trading floors, in particular. These management techniques are precise about the risk distributions and the characteristics of each security.

Risk management can increase shareholders' value if the risk management can reduce transaction costs, taxes, or affect investment decisions. With real options, the cost of capital can change, the strategic investments can be affected by default and other factors, and the firm value can be affected.

NOTES

1. Since we use the same stock prices as the delta-normal method, we have the same variance–covariance matrix, which means that we have the same betas.
2. The example is taken from Thomas S.Y. Ho, Allen Abrahamson, and Mark Abbott 1996 "Value at Risk of a Bank's Balance Sheet," *International Journal of Theoretical and Applied Finance*, vol. 2, no. 1, January 1999.
3. Jorion, P., 2001, *Value at Risk*, 2nd edition, McGraw Hill. "For more information, see"
4. 12.47086 is not equal to the sum of three numbers (4.88495, 4.05485, 6.60260) because of diversification effect.

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